

Compound Interest

Lecture 3 Section 10.3

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Fri, Jan 19, 2018

- 1 Compound Interest
- 2 The Rule of 72
- 3 The Compound Interest Formula
- 4 The Effective Interest Rate
- 5 Certificates of Deposit
- 6 Assignment

Outline

- 1 Compound Interest
- 2 The Rule of 72
- 3 The Compound Interest Formula
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Compound Interest

Definition (Compound Interest)

When a loan is based on **compound interest**, interest is paid on the principal *and on all interest accrued so far*.

Definition (Compounding Period)

The **compounding period** is the length of time over which the interest is computed when it is compounded.

- The compounding period is usually expressed as the number of such periods per year.

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Compound Interest

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Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00

Compound Interest

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- A principal of \$1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50

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- A principal of \$1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50
3	1102.50	55.13	1157.63

Compound Interest

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- A principal of \$1000 is invested at 5% interest compounded annually for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50
3	1102.50	55.13	1157.63
4	1157.63	57.88	1215.51

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
- Find the future value.

Compound Interest

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- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
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Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90

Compound Interest

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- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82

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- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
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Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42

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2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42
6	1131.42	28.29	1159.71

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3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42
6	1131.42	28.29	1159.71
7	1159.71	28.99	1188.70

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Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded **semi-annually** for 4 years.
- Find the future value.

Period	Principal	Interest	Balance
1	1000.00	25.00	1025.00
2	1025.00	25.63	1050.63
3	1050.63	26.27	1076.90
4	1076.90	26.92	1103.82
5	1103.82	27.60	1131.42
6	1131.42	28.29	1159.71
7	1159.71	28.99	1188.70
8	1188.70	29.72	1218.42

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The Rule of 72

Definition (The Rule of 72)

The **Rule of 72** says that an interest rate of $r\%$ will double the value of an investment in approximately $72/r$ years.

- An effective rate of 9% will double the value of the investment in 8 years.
- By how much will the investment (at 9%) grow in 16 years?
- How long will it take for the future value to grow to 8 times the principal? 16 times?

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The Compound Interest Formula

- If the interest is compounded **annually**, then

$$F = P(1 + r)^t.$$

- If the interest is compounded **k times a year**, then

$$F = P \left(1 + \frac{r}{k} \right)^{kt}.$$

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded monthly for 4 years.
- Find the future value.

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded monthly for 4 years.
- Find the future value.
- Find the future value if it is compounded quarterly.

Compound Interest

Example (Compound Interest)

- A principal of \$1000 is invested at 5% interest compounded monthly for 4 years.
- Find the future value.
- Find the future value if it is compounded quarterly.
- Find the future value if it is compounded monthly for 50 years vs. simple interest for 50 years.

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The Effective Interest Rate

Definition (Effective Interest Rate)

For a given compound interest rate, the **effective interest rate** r_{eff} , or **annual percentage yield (APY)**, is the interest rate that produces the same yield when compounded annually.

- Given an interest rate r compounded k time a year, the effective rate is given by

$$\text{APY} = \left(1 + \frac{r}{k}\right)^k - 1.$$

The Effective Interest Rate

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For a given compound interest rate, the **effective interest rate** r_{eff} , or **annual percentage yield (APY)**, is the interest rate that produces the same yield when compounded annually.

- Given an interest rate r compounded k time a year, the effective rate is given by

$$\text{APY} = \left(1 + \frac{r}{k}\right)^k - 1.$$

- This is the same as computing the interest earned on \$1.00 in one year.

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly
 - Monthly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly
 - Monthly
 - Daily

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly
 - Monthly
 - Daily
 - Hourly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly
 - Monthly
 - Daily
 - Hourly
 - Minutely

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly
 - Monthly
 - Daily
 - Hourly
 - Minutely
 - Secondly

The Effective Interest Rate

Example (Effective Interest Rate)

- Find the effective interest rate, or APY, of 12% compounded
 - Annually
 - Semiannually
 - Quarterly
 - Monthly
 - Daily
 - Hourly
 - Minutely
 - Secondly
- Do you notice a trend?

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Certificates of Deposit

Definition (Certificate of Deposit)

A **certificate of deposit (CD)** is an investment, usually through a bank, for a fixed period of time at a fixed compound interest rate.

- There is typically a penalty for early withdrawal. The penalty is usually equal to 6 months of interest.

Certificates of Deposit

Example (Certificate of Deposit)

- What is the future value of a 5-year CD earning 2% interest, compounded quarterly?

Certificates of Deposit

Example (Certificate of Deposit)

- What is the future value of a 5-year CD earning 2% interest, compounded quarterly?
- Suppose that at the end of one year, the rate on new CDs is 3%. Should the investor cash in the CD earning 1%, pay the penalty of 6 months' interest, and reinvest the difference in a new CD earning 3%?

Certificates of Deposit

Example (Certificate of Deposit)

- What is the future value of a 5-year CD earning 2% interest, compounded quarterly?
- Suppose that at the end of one year, the rate on new CDs is 3%. Should the investor cash in the CD earning 1%, pay the penalty of 6 months' interest, and reinvest the difference in a new CD earning 3%?
- What if the penalty were 12 months' interest?

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Assignment

Assignment

- Chapter 10: Exercises 27, 28, 31, 32, 41, 42, 43, 44, 70.